Problem: Let E be a subset of $\mathbb{R}$. We define the boundary of E to be

$$\partial E := \{x \in \mathbb{R} \setminus E : \text{ for all } \epsilon > 0,\ E \cap (x - \epsilon, x + \epsilon) \neq \emptyset \ \}$$

Does there exist a set E such that $\lambda_1(\partial E) > 0$?

Problem: Let K be a subset of $\mathbb{R}^2$ with the property that a unit line segment can be *continuously* rotated $180°$ within K. Is it true that $\lambda_2(K) > 0$?

Problem: For any subset E of $\mathbb{R}$, define the difference set of E to be

$$\mathcal{D}(E) := \{x - y \in \mathbb{R} : x \in E \text{ and } y \in E\}$$

If $\lambda_1(E) = 0$, does $\lambda_1(\mathcal{D}(E)) = 0$?

Problem: Let $-\infty < a < b < \infty$ and $f : [a, b] \to \mathbb{R}$ be continuous. Define

$$\text{graph}(f) = \{(x, y) \in \mathbb{R}^2 : \; x \in [a, b] \text{ and } f(x) = y\}$$

Does $\lambda_2(\text{graph}(f)) = 0$?

Problem: Let E be a subset of $\mathbb{R}^2$. Define for each natural number n the set

$$\mathcal{O}_n = \{x \in \mathbb{R}^2 : d(x, E) < 1/n\},$$

where $d(x, E)$ is the distance between x and E. Is it true that $\lambda_2(E) = \lim_{n\to\infty} \lambda_2(\mathcal{O}_n)$?

Problem: Let $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} 1 & x \in [0,1] \\ 0 & x \notin [0,1] \end{cases}.$$

Does there exists a continuous function $g : \mathbb{R} \to \mathbb{R}$ and a set $Z \subset \mathbb{R}$ such that $\lambda_1(Z) = 0$ and

$$g(x) = f(x) \quad \text{for all } x \in \mathbb{R} \setminus Z?$$