Teaching in Math - M398T Handout

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Warm up: Calculate following integrals:

$$\int_0^1 xe^x dx \quad \text{and} \quad \iiint_{B(0,1)} |x|^4 dV$$

where $B(0,1)$ is the unit ball in $\mathbb{R}^3$ centered at 0.

Problem: First, define $\varphi : \mathbb{R}^3 \to \mathbb{R}$ and $\vec{v} : \mathbb{R}^3 \to \mathbb{R}^3$ by

$$\varphi(x_1, x_2, x_3) := 1 - (x_1^2 + x_2^2 + x_3^2)$$

$$\vec{v}(x_1, x_2, x_3) := \left(x_1^3 + 2x_1x_2^2 + 2x_1x_3^2 \, , \, x_2^3 + 2x_2x_3^2 \, , \, x_3^3 \right)$$

Calculate the integral

$$\iiint_{B(0,1)} \varphi \, \mathrm{div}\vec{v} \, dV$$

where $B(0,1)$ is the unit sphere centered 0 in $\mathbb{R}^3$.

Hint: what is $(x_1^2 + x_2^2 + x_3^2)^2$?

Formulas

$$\int u dv = uv - \int v du$$

$$\iiint_{B(0,R)} f dx = \int_0^R \iint_{\partial B(0,r)} f dS dr$$

$$\iiint_\Omega \varphi \ \mathrm{div}(\vec{v}) dx = \iint_{\partial\Omega} \varphi \vec{v} \cdot \vec{n} dS - \iiint_\Omega \vec{v} \cdot \nabla\varphi dx$$